

Charge symmetry breaking in Λ hypernuclei revisited

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Abstract

The large charge symmetry breaking (CSB) implied by the Λ binding energy difference $\Delta B_{\Lambda}^4(0_{\text{g.s.}}^+) \equiv B_{\Lambda}(^4_{\Lambda}\text{He}) - B_{\Lambda}(^4_{\Lambda}\text{H}) = 0.35 \pm 0.06$ MeV of the $A = 4$ mirror hypernuclei ground states, determined from emulsion studies, has defied theoretical attempts to reproduce it in terms of CSB in hyperon masses and in hyperon-nucleon interactions, including one pion exchange arising from $\Lambda - \Sigma^0$ mixing. Using a schematic strong-interaction $\Lambda N \leftrightarrow \Sigma N$ coupling model developed by Akaishi and collaborators for s -shell Λ hypernuclei, we revisit the evaluation of CSB in the $A = 4$ Λ hypernuclei and extend it to p -shell mirror Λ hypernuclei. The model yields values of $\Delta B_{\Lambda}^4(0_{\text{g.s.}}^+) \sim 0.25$ MeV. Smaller size and mostly negative p -shell binding energy differences are calculated for the $A = 7 - 10$ mirror hypernuclei, in rough agreement with the few available data. CSB is found to reduce by almost 30 keV the 110 keV $^{10}_{\Lambda}\text{B}$ g.s. doublet splitting anticipated from the hyperon-nucleon strong-interaction spin dependence, thereby explaining the persistent experimental failure to observe the $2_{\text{exc}}^- \rightarrow 1_{\text{g.s.}}^-$ γ -ray transition.

Keywords:

charge symmetry breaking, hypernuclei, effective interactions in hadronic systems, shell model

1. Introduction

Charge symmetry breaking (CSB) in nuclear physics is primarily identified by considering the difference between nn and pp scattering lengths, or the binding-energy difference between the mirror nuclei ^3H and ^3He [1]. In these nuclei, about 70 keV out of the Coulomb-dominated 764 keV binding-energy difference is commonly attributed to CSB which can be explained either by $\rho^0\omega$ mixing in one-boson exchange models of the NN interaction,

or by considering $N\Delta$ intermediate-state mass differences in models limited to pseudoscalar meson exchanges [2].

In Λ hypernuclei, in contrast, CSB appears to be considerably stronger, judging by the binding-energy difference $\Delta B_\Lambda^4(0_{\text{g.s.}}^+) = 0.35 \pm 0.06$ MeV deduced from the level diagram of the mirror hypernuclei (${}^4_\Lambda\text{H}$, ${}^4_\Lambda\text{He}$) in Fig. 1. A very recent measurement of ${}^4_\Lambda\text{H} \rightarrow {}^4\text{He} + \pi^-$ decay at MAMI [5] reduces $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ to 0.27 ± 0.10 MeV, consistent with its emulsion value [3]. Fig. 1 also suggests that $\Delta B_\Lambda^4(1_{\text{exc}}^+)$ is almost as large as $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$. However, the deduction of the 1^+ excitation energy in ${}^4_\Lambda\text{He}$ from the 1.15 MeV $1_{\text{exc}}^+ \rightarrow 0_{\text{g.s.}}^+$ γ -ray transition [6] is not as firm as the one for ${}^4_\Lambda\text{H}$ [4]. In passing we mention the weak 1.42 MeV γ -ray transition reported in Ref. [7] that would imply almost no CSB splitting of the 1_{exc}^+ states if its assignment to ${}^4_\Lambda\text{He}$ gets confirmed.

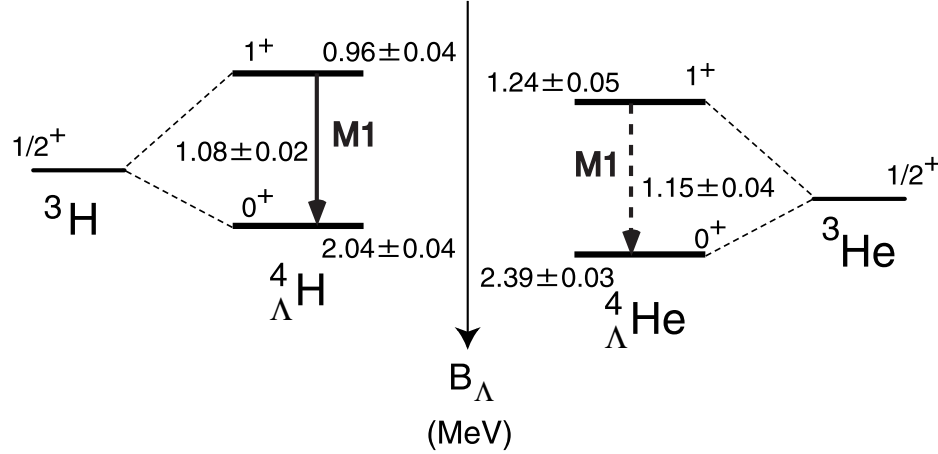


Figure 1: Level diagram of mirror hypernuclei (${}^4_\Lambda\text{H}$, ${}^4_\Lambda\text{He}$) obtained by adding a Λ hyperon to the mirror nuclei (${}^3\text{H}$, ${}^3\text{He}$). The Λ separation energies, also loosely termed Λ binding energies (B_Λ in MeV), are taken from emulsion work [3]. Figure adapted from Ref. [4].

The large ΔB_Λ^4 values reported for both $0_{\text{g.s.}}^+$ and 1_{exc}^+ states have defied theoretical attempts to explain these differences in terms of hadronic or quark CSB mechanisms within four-body calculations [8, 9, 10, 11, 12]. Meson mixing, including $\rho^0\omega$ mixing which explains CSB in the $A = 3$ nuclei, gives only small *negative* contributions about -30 and -10 keV for $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ and $\Delta B_\Lambda^4(1_{\text{exc}}^+)$, respectively [9]. CSB contributions to $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ from one- and two-pion exchange interactions in $YNNN$ coupled-channel calculations [10, 11] with hyperons $Y = \Lambda, \Sigma$ amount to as much as 100 keV; this holds

for the OBE-based Nijmegen NSC97 models [13] which are widely used in Λ -hypernuclear structure calculations.¹

Binding energies of ground states in p -shell mirror Λ hypernuclei, determined from emulsion studies [3], suggest much weaker CSB effects than for $A = 4$, with values of $\Delta B_\Lambda^A \equiv B_\Lambda(A, I, I_z) - B_\Lambda(A, I, -I_z)$ ($I_z > 0$) consistent with zero for $A = 8$ and somewhat negative beyond [3]. Accommodating ΔB_Λ values in the p shell with ΔB_Λ^4 by using reasonable phenomenological CSB interactions is impossible, as demonstrated in recent four-body cluster-model calculations of p -shell Λ hypernuclei [15]. This difficulty may be connected to the absence of explicit $\Lambda N \leftrightarrow \Sigma N$ coupling in these cluster-model calculations, given that such explicit coupling was shown to generate non-negligible CSB contributions to $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ [12]. It is our purpose in this note to use a schematic $\Lambda N \leftrightarrow \Sigma N$ coupling model, proposed by Akaishi et al. [16, 17] for s -shell Λ hypernuclei and extended by Millener [18] to the p shell, for calculating ΔB_Λ values in both s and p shells, thereby making predictions on CSB effects in p -shell Λ hypernuclei consistently with a relatively sizable value of $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$.

The paper is organized as follows. In Sect. 2, we update the original treatment by Dalitz and Von Hippel [19] of the $\Lambda - \Sigma^0$ mixing mechanism for generating CSB one-pion exchange contributions in Λ hypernuclei, linking it to the strong-interaction $\Lambda N \leftrightarrow \Sigma N$ coupling model employed in this work. Our CSB calculations for the $A = 4$ hypernuclei are sketched in Sect. 3 and their results are compared with those reported in several $YNNN$ four-body calculations [9, 10, 11, 12]. Finally, CSB contributions in p -shell mirror Λ hypernuclei, evaluated here for the first time, are reported in Sect. 4.

2. Pionic CSB contributions in Λ hypernuclei

The $I = 0$ isoscalar nature of the Λ hyperon forbids it to emit or absorb a single pion, and hence there is no one-pion exchange (OPE) contribution to the ΛN strong interaction. However, by allowing for $\Lambda - \Sigma^0$ mixing in

¹Contradictory statements were made in Refs. [8, 10] on the ability of the earlier Nijmegen model NSC89 [14] to reproduce $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$. We note that the strength of CSB contributions in this work, on p. 2236, is inflated erroneously by a factor g_{NNM} , about 3.7 for pion exchange, which might have propagated into some of the calculations claiming to have resolved the CSB puzzle for $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ using NSC89.

SU(3), a CSB OPE contribution arises [19] with $\Lambda\Lambda\pi$ coupling constant

$$g_{\Lambda\Lambda\pi} = -2 \frac{\langle \Sigma^0 | \delta M | \Lambda \rangle}{M_{\Sigma^0} - M_{\Lambda}} g_{\Lambda\Sigma\pi} = -0.0297 g_{\Lambda\Sigma\pi}, \quad (1)$$

where the matrix element of the mass mixing operator δM is given by

$$\langle \Sigma^0 | \delta M | \Lambda \rangle = \frac{1}{\sqrt{3}} (M_{\Sigma^0} - M_{\Sigma^+} + M_p - M_n). \quad (2)$$

The resulting CSB OPE potential is given by

$$V_{\Lambda N}^{\text{CSB}}(\text{OPE}) = -0.0242 \tau_{Nz} \frac{f_{NN\pi}^2}{4\pi} \frac{m_\pi}{3} [\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N + T(r) S(\hat{r}; \vec{\sigma}_\Lambda, \vec{\sigma}_N)] Y(r), \quad (3)$$

where the z component of the isospin Pauli matrix $\vec{\tau}_N$ assumes the values $\tau_{Nz} = \pm 1$ on protons and neutrons, respectively, $Y(r) = \exp(-m_\pi r)/(m_\pi r)$ is a Yukawa form, and the tensor contribution is specified by

$$T(r) = 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}, \quad S(\hat{r}; \vec{\sigma}_\Lambda, \vec{\sigma}_N) = 3\vec{\sigma}_\Lambda \cdot \hat{r} \vec{\sigma}_N \cdot \hat{r} - \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N. \quad (4)$$

In Eq. (3), the transition $g_{\Lambda\Sigma\pi} \rightarrow f_{NN\pi}$ was made in accordance with NSC models, using $f_{NN\pi}^2/4\pi = 0.0740$ from NSC89, Table IV in Ref [14].

Since Pauli-spin $S_{pp} = 0$ and $S_{nn} = 0$ hold in ${}^4_\Lambda\text{He}(0^+_{\text{g.s.}})$ and in ${}^4_\Lambda\text{H}(0^+_{\text{g.s.}})$, respectively, the CSB potential (3) which is linear in the nucleon spin gives no contribution from these same-charge nucleons. Therefore $\tau_{Nz} = \mp 1$ owing to the odd nucleon in these hypernuclei, respectively, and since $\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N = -3$ holds for this odd nucleon, one gets a *positive* ΔB_Λ contribution from the central spin-spin part which provides the only nonvanishing contribution for simple $L = 0$ wavefunctions. For the $0^+_{\text{g.s.}}$ wavefunction used by Dalitz and Von Hippel [19], and updating the values of the coupling constants used in their work to those used here, one gets $\Delta B_\Lambda^{\text{OPE}}(0^+_{\text{g.s.}}) \approx 95$ keV, a substantial single contribution with respect to $\Delta B_\Lambda^{\text{exp}}(0^+_{\text{g.s.}}) = 350 \pm 60$ keV.

The $\Lambda - \Sigma^0$ mixing mechanism gives rise also to a variety of (e.g. ρ) meson exchanges other than OPE. In baryon-baryon models that consider *explicitly* the strong-interaction $\Lambda N \leftrightarrow \Sigma N$ coupling, the matrix element of $V_{\Lambda N}^{\text{CSB}}$ is related to a suitably chosen strong-interaction isospin $I_{NY} = 1/2$ matrix element $\langle N\Sigma | V | N\Lambda \rangle$ by generalizing Eq. (1):

$$\langle N\Lambda | V_{\Lambda N}^{\text{CSB}} | N\Lambda \rangle = -0.0297 \tau_{Nz} \frac{1}{\sqrt{3}} \langle N\Sigma | V | N\Lambda \rangle, \quad (5)$$

where the isospin Clebsch-Gordan coefficient $1/\sqrt{3}$ accounts for the $N\Sigma^0$ amplitude in the $I_{NY} = 1/2$ $N\Sigma$ state, and the space-spin structure of this $N\Sigma$ state is taken identical with that of the $N\Lambda$ state sandwiching $V_{\Lambda N}^{\text{CSB}}$.

3. CSB in the $A = 4$ hypernuclei

Table 1: $\Lambda\Sigma$ s -shell matrix elements $\bar{V}_{\Lambda\Sigma}^{0s}$ and $\Delta_{\Lambda\Sigma}^{0s}$, downward energy shifts $\delta E_{\downarrow}(J^\pi)$ calculated for the $A = 4$ hypernuclear states, the resulting $\Delta E_{\Lambda\Sigma}(0_{\text{g.s.}}^+ - 1_{\text{exc}}^+)$ in models $(\Lambda\Sigma)_{\text{e,f}}$ [18] and the full excitation energy $\Delta E(0_{\text{g.s.}}^+ - 1_{\text{exc}}^+)$ [10, 16, 17], all in MeV.

NSC97 model	$\bar{V}_{\Lambda\Sigma}^{0s}$	$\Delta_{\Lambda\Sigma}^{0s}$	$\delta E_{\downarrow}(0_{\text{g.s.}}^+)$	$\delta E_{\downarrow}(1_{\text{exc}}^+)$	$\Delta E_{\Lambda\Sigma}$	$\Delta E(0_{\text{g.s.}}^+ - 1_{\text{exc}}^+)$		
	as calculated in models $(\Lambda\Sigma)_{\text{e,f}}$ [18]					[10]	[16]	[17]
NSC97 _e	2.96	5.09	0.574	0.036	0.539	0.75	0.89	1.13
NSC97 _f	3.35	5.76	0.735	0.046	0.689	1.10	1.48	1.51

Following hyperon-core calculations of s -shell Λ hypernuclei by Akaishi et al. [16] we use G -matrix YN effective interactions derived from NSC97 models to calculate CSB contributions from Eq. (5). The $\Lambda\Sigma 0s_N 0s_Y$ effective interaction $V_{\Lambda\Sigma}$ is given in terms of a spin-dependent central interaction

$$V_{\Lambda\Sigma} = (\bar{V}_{\Lambda\Sigma} + \Delta_{\Lambda\Sigma} \vec{s}_N \cdot \vec{s}_Y) \sqrt{4/3} \vec{t}_N \cdot \vec{t}_{\Lambda\Sigma}, \quad (6)$$

where $\vec{t}_{\Lambda\Sigma}$ converts a Λ to Σ in isospace. The s -shell matrix elements $\bar{V}_{\Lambda\Sigma}^{0s}$ and $\Delta_{\Lambda\Sigma}^{0s}$ are listed in Table 1 for two such G -matrix models denoted $(\Lambda\Sigma)_{\text{e,f}}$. Also listed are the calculated downward energy shifts $\delta E_{\downarrow}(J^\pi)$ defined by $\delta E_{\downarrow}(J^\pi) = v^2(J^\pi)/(80 \text{ MeV})$, where the $0s_N^3 0s_Y$ matrix elements $v(J^\pi)$ for $A = 4$ are given in terms of $\Lambda\Sigma$ two-body matrix elements by

$$v(0_{\text{g.s.}}^+) = \bar{V}_{\Lambda\Sigma}^{0s} + \frac{3}{4} \Delta_{\Lambda\Sigma}^{0s}, \quad v(1_{\text{exc}}^+) = \bar{V}_{\Lambda\Sigma}^{0s} - \frac{1}{4} \Delta_{\Lambda\Sigma}^{0s}. \quad (7)$$

We note that the diagonal $0s_N 0s_\Sigma$ interaction matrix elements have little effect in this coupled-channel model because of the large energy denominators of order $M_\Sigma - M_\Lambda \approx 80 \text{ MeV}$ with which they appear. Finally, by listing $\Delta E_{\Lambda\Sigma}(0_{\text{g.s.}}^+ - 1_{\text{exc}}^+)$ from Refs. [16, 17] we demonstrate the sizable contribution of $\Lambda\Sigma$ coupling to the excitation energy $\Delta E(0_{\text{g.s.}}^+ - 1_{\text{exc}}^+) \approx 1.1 \text{ MeV}$ deduced from the γ -ray transition energies marked in Fig. 1. For comparison, the full $\Delta E(0_{\text{g.s.}}^+ - 1_{\text{exc}}^+)$ in these $(\Lambda\Sigma)_{\text{e,f}}$ models, and as calculated by Nogga [10] using the underlying Nijmegen models NSC97_{e,f}, are also listed in the table.

Having discussed the effect of strong-interaction $\Lambda\Sigma$ coupling, we now discuss the CSB splittings $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ and $\Delta B_\Lambda^4(1_{\text{exc}}^+)$. Results of our $\Lambda\Sigma$ coupling model calculations, using Eq. (5) for one of several contributions, are listed in the last two lines of Table 2, preceded by results obtained in other models within genuine four-body calculations [9, 10, 11, 12]. Partial contributions to $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ are listed in columns 2–5, whereas for $\Delta B_\Lambda^4(1_{\text{exc}}^+)$ only its total value is listed.

Table 2: Calculated CSB contributions to $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ and total values of $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ and $\Delta B_\Lambda^4(1_{\text{exc}}^+)$, in keV, from several model calculations of the $A = 4$ hypernuclei. Recall that $\Delta B_\Lambda^{\text{exp}}(0_{\text{g.s.}}^+) = 350 \pm 60$ keV [3].

${}^4_\Lambda\text{He}-{}^4_\Lambda\text{H}$ model	$P_\Sigma(\%)$ $0_{\text{g.s.}}^+$	ΔT_{YN} $0_{\text{g.s.}}^+$	ΔV_C $0_{\text{g.s.}}^+$	ΔV_{YN} $0_{\text{g.s.}}^+$	ΔB_Λ^4 $0_{\text{g.s.}}^+$	ΔB_Λ^4 1_{exc}^+
ΛNNN [9]	—	—	−42	91	49	−61
NSC97 _e [10]	1.6	47	−16	44	75	−10
NSC97 _f [11]	1.8				100	−10
NLO chiral [12]	2.1	55	−9	—	46	
$(\Lambda\Sigma)_e$ [present]	0.72	39	−45	232	226	30
$(\Lambda\Sigma)_f$ [present]	0.92	49	−46	263	266	39

All of the models listed in Table 2 except for [9] include $\Lambda\Sigma$ coupling, with $0_{\text{g.s.}}^+$ ΣNNN admixture probabilities $P_\Sigma \approx P_{\Sigma^\pm} + P_{\Sigma^0}$ in $({}^4_\Lambda\text{He}, {}^4_\Lambda\text{H})$ respectively, and $P_{\Sigma^\pm} \approx \frac{2}{3}P_\Sigma$. The 1_{exc}^+ ΣNNN admixtures (unlisted) are considerably weaker than the listed $0_{\text{g.s.}}^+$ admixtures. Charge asymmetric kinetic-energy contributions to ΔB_Λ , dominated by ΣN intermediate-state mass differences, are marked ΔT_{YN} in the table. In the present $\Lambda\Sigma$ coupling model these are given for the $0_{\text{g.s.}}^+$ by [10]

$$\Delta T_{YN}(0_{\text{g.s.}}^+) \approx \frac{2}{3} P_\Sigma (M_{\Sigma^-} - M_{\Sigma^+}), \quad (8)$$

yielding as much as 50 keV, in agreement with those four-body calculations where such mass differences were introduced [10, 11, 12]. The next column in the table, $\Delta V_C = \Delta V_C^\Lambda + \Delta V_C^\Sigma$, addresses contributions arising from nuclear-core Coulomb energy modifications induced by the hyperons. ΔV_C^Λ is negative, its size ranges from less than 10 keV [10, 12] to about 40 keV [9]. ΔV_C^Σ which accounts for $\Sigma^\pm p$ Coulomb energies in the ΣNNN admixed components is also negative and uniformly small with size of a few keV at most.

The values assigned to ΔV_C in the $\Lambda\Sigma$ model use values from Ref. [9] for ΔV_C^Λ and the estimate $\Delta V_C^\Sigma \approx -\frac{2}{3}P_\Sigma E_C(^3\text{He})$ for ΔV_C^Σ , where $E_C(^3\text{He}) = 644$ keV is the Coulomb energy of ^3He .

The next contribution, ΔV_{YN} , is derived from $V_{\Lambda N}^{\text{CSB}}$. No ΔV_{YN} contributions are available from the coupled channels calculation by Hiyama et al. [20] (not listed here) and also from the recent chiral-model calculation in which CSB contributions are disregarded [12] in order to remain consistent with EFT power counting rules that exclude CSB from the NLO chiral version of the YN interaction [21]. With the exception of the purely ΛNNN four-body calculation of Ref. [9], all those models for which a nonzero value is listed in the table effectively used Eq. (5) to evaluate $\Delta V_{YN}(0_{\text{g.s.}}^+)$. This ensures that meson exchanges arising from $\Lambda - \Sigma^0$ mixing beyond OPE are also included in the calculated CSB contribution. Generally, the CSB potential contribution $\Delta V_{YN}(0_{\text{g.s.}}^+)$ is not linked in any simple model-independent way to the Σ admixture probability $P_\Sigma(0_{\text{g.s.}}^+)$. For example, the calculations using NSC97 [10, 11] produce too little CSB contributions, whereas the present $\Lambda\Sigma$ model, in spite of its weaker Σ admixtures, gives sizable contributions which essentially resolve the CSB puzzle in the $0_{\text{g.s.}}^+$ of the $A = 4$ hypernuclei. Indeed, using a typical $\Lambda\Sigma$ strong-interaction matrix element $\langle N\Sigma|V(0_{\text{g.s.}}^+)|N\Lambda\rangle \sim 7$ MeV in Eq. (5) one obtains $P_\Sigma(0_{\text{g.s.}}^+) = 0.77\%$ and a CSB contribution of 240 keV to $\Delta B_\Lambda(0_{\text{g.s.}}^+)$; this CSB contribution is proportional to $\sqrt{P_\Sigma}$ in the present $\Lambda\Sigma$ model.

The resulting values of $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$ listed in Table 2 are smaller than 100 keV within the calculations presented in Refs. [9, 10, 11, 12], leaving the $A = 4$ CSB puzzle unresolved, while being larger than 200 keV in the present $\Lambda\Sigma$ model and thereby getting considerably closer to the experimentally reported $0_{\text{g.s.}}^+$ CSB splitting. The main difference between these two groups of calculations arises from the difference in the CSB potential contributions $\Delta V_{YN}(0_{\text{g.s.}}^+)$. A similarly large difference also appears between the CSB potential *negative* contributions $\Delta V_{YN}(1_{\text{exc}}^+)$ in the calculations of Refs. [9, 10, 11] and the *positive* contributions $\Delta V_{YN}(1_{\text{exc}}^+)$ in the present $\Lambda\Sigma$ model, resulting in large but different $\Delta B_\Lambda^4(0_{\text{g.s.}}^+) - \Delta B_\Lambda^4(1_{\text{exc}}^+)$ values, about 200 keV in the present $\Lambda\Sigma$ model and about 100 keV for all other calculations [9, 10, 11]. A common feature of all CSB model calculations so far is that none of them is able to generate values in excess of 50 keV for $\Delta B_\Lambda^4(1_{\text{exc}}^+)$.

A direct comparison between the NCS97 models and the present $\Lambda\Sigma$ model is not straightforward because the $\Lambda\Sigma$ coupling in NSC97 models is

dominated by tensor components, whereas no tensor components appear in present $\Lambda\Sigma$ model. It is worth noting, however, that the ρ exchange contribution to the matrix element $\langle N\Sigma|V|N\Lambda\rangle$ in Eq. (5) is of opposite sign to that of OPE for the tensor $\Lambda\Sigma$ coupling which dominates in NSC models, leading to cancellations, whereas both ρ exchange and OPE contribute constructively in the present central $\Lambda\Sigma$ coupling model in agreement with the calculation by Coon et al. [9] which also has no tensor components.² This point deserves further study by modeling various input YN interactions in future four-body calculations.

4. CSB in p -shell hypernuclei

Several few-body cluster-model calculations, of the $A = 7$, $I = 1$ isotriplet [22] and the $A = 10$, $I = \frac{1}{2}$ isodoublet [23], have considered the issue of CSB contributions to Λ binding energy differences of p -shell mirror hypernuclei. It was verified in these calculations that the introduction of a ΛN phenomenological CSB interaction fitted to ΔB_Λ^4 , for both $0_{\text{g.s.}}^+$ and 1_{exc}^+ states, failed to reproduce the observed ΔB_Λ^4 values in these p -shell hypernuclei; in fact, it only aggravated the discrepancy between experiment and calculations. Although it is possible to reproduce the observed values by introducing additional CSB components that hardly affect ΔB_Λ^4 , this prescription lacks any physical origin and is therefore questionable, as acknowledged very recently by Hiyama [15]. Here we explore p -shell CSB contributions, extending the NSC97e model $0s_N 0s_Y$ effective interactions considered in Sect. 3, by providing $(\Lambda\Sigma)_e$ $0p_N 0s_Y$ central-interaction matrix elements which are consistent with the role $\Lambda N \leftrightarrow \Sigma N$ coupling appears to play in a shell-model reproduction of hypernuclear γ -ray transition energies [24]:

$$\bar{V}_{\Lambda\Sigma}^{0p} = 1.45, \quad \Delta_{\Lambda\Sigma}^{0p} = 3.04 \quad (\text{in MeV}). \quad (9)$$

These p -shell matrix elements are smaller by roughly a factor of two from the corresponding s -shell matrix elements in Table 1, reflecting a reduced weight, about $1/2$, with which the dominant relative s -wave matrix elements of V_{NY} appear in the p shell. This suggests that Σ admixtures which are quadratic in these matrix elements, are weaker roughly by a factor of 4 with respect

²It is worth noting that the ρ exchange CSB contribution calculated in Ref. [9] is of the same sign and remarkably stronger than the OPE CSB contribution.

to the s -shell calculation, and also smaller CSB interaction contributions in the p shell with respect to those in the $A = 4$ hypernuclei, although only by a factor of 2. To evaluate these CSB contributions, instead of applying the one-nucleon or nucleon-hole expression (5) valid in the s shell, we use in the p shell the general multi-nucleon expression for $V_{\Lambda N}^{\text{CSB}}$ obtained by summing over p -shell nucleons:

$$V_{\Lambda N}^{\text{CSB}} = -0.0297 \frac{1}{\sqrt{3}} \sum_j (\bar{V}_{\Lambda\Sigma}^{0p} + \Delta_{\Lambda\Sigma}^{0p} \vec{s}_j \cdot \vec{s}_Y) \tau_{jz}. \quad (10)$$

Results of applying the present $(\Lambda\Sigma)_e$ coupling model to several pairs of g.s. levels in p -shell hypernuclear isomultiplets are given in Table 3. All pairs except for $A = 7$ are mirror hypernuclei identified in emulsion [3] where binding energy systematic uncertainties are largely canceled out in forming the listed $\Delta B_{\Lambda}^{\text{exp}}$ values. For $A = 7$ we compensated for the unavailability of a reliable $B_{\Lambda}({}^7_{\Lambda}\text{He})$ value from emulsion by replacing it with $B_{\Lambda}({}^7_{\Lambda}\text{Li}^*)$, established by observing the 3.88 MeV γ -ray transition ${}^7_{\Lambda}\text{Li}^* \rightarrow \gamma + {}^7_{\Lambda}\text{Li}$ [28]. Recent B_{Λ} values determined in electroproduction experiments at JLab for ${}^7_{\Lambda}\text{He}$ [29, 30], ${}^9_{\Lambda}\text{Li}$ [31] and ${}^{10}_{\Lambda}\text{Be}$ [30] were not used for lack of similar data on their mirror partners.

Table 3: CSB contributions to $\Delta B_{\Lambda}^{\text{calc}}(\text{g.s.})$ values in p -shell hypernuclear isomultiplets, using the $(\Lambda\Sigma)_e$ coupling model. The s -shell contributions to $\Delta B_{\Lambda}^4(0_{\text{g.s.}}^+)$ from Table 2 are also listed for comparison.

${}^A_{\Lambda}Z_{>} - {}^A_{\Lambda}Z_{<}$ pairs	I, J^{π}	P_{Σ} (%)	ΔT_{YN} (keV)	ΔV_C (keV)	ΔV_{YN} (keV)	$\Delta B_{\Lambda}^{\text{calc}}$ (keV)	$\Delta B_{\Lambda}^{\text{exp}}[3]$ (keV)
${}^4_{\Lambda}\text{He} - {}^4_{\Lambda}\text{H}$	$\frac{1}{2}, 0^+$	0.72	39	-45	232	226	+350±60
${}^7_{\Lambda}\text{Be} - {}^7_{\Lambda}\text{Li}^*$	$1, \frac{1}{2}^+$	0.12	3	-70 [22]	50	-17	-100±90
${}^8_{\Lambda}\text{Be} - {}^8_{\Lambda}\text{Li}$	$\frac{1}{2}, 1^-$	0.20	11	-81 [26]	119	+49	+40±60
${}^9_{\Lambda}\text{B} - {}^9_{\Lambda}\text{Li}$	$1, \frac{3}{2}^+$	0.23	10	-145 [27]	81	-54	-210±220
${}^{10}_{\Lambda}\text{B} - {}^{10}_{\Lambda}\text{Be}$	$\frac{1}{2}, 1^-$	0.053	3	-156 [27]	17	-136	-220±250

The Σ admixture percentages P_{Σ} in Table 3 follow from $\Lambda\Sigma$ strong-interaction contributions to p -shell hypernuclear g.s. energies computed in Ref. [24], and the associated CSB kinetic-energy contributions ΔT_{YN} were calculated using a straightforward generalization of Eq. (8). These contributions, of order 10 keV and less, are considerably weaker than the ΔT_{YN}

contributions to ΔB_Λ^4 listed in Table 2, reflecting weaker Σ admixtures in the p shell as discussed following Eq. (9). The Coulomb-induced contributions ΔV_C are dominated by their ΔV_C^Λ components which were taken from Hiyama’s cluster-model calculations [22, 26] for $A = 7, 8$ and from Millener’s shell-model calculations [27] for $A = 9, 10$. The shell-model estimate of -156 keV adopted here for $A = 10$ is somewhat smaller than the -180 keV cluster-model result [23]. The ΔV_C^Σ components are negligible, with size of 1 keV at most (for $A = 8, 9$). ΔV_C^Λ is always negative, as expected from the increased Coulomb repulsion owing to the increased proton separation energy in the Λ hypernucleus with respect to its core. The sizable negative p -shell ΔV_C contributions, in distinction from their secondary role in forming the total $\Delta B_\Lambda^4(0_{\text{g.s.}}^+)$, exceed in size the positive p -shell ΔV_{YN} contributions by a large margin beginning with $A = 9$, thereby resulting in clearly negative values of $\Delta B_\Lambda^4(\text{g.s.})$.

The CSB ΔV_{YN} contributions listed in Table 3 were calculated using weak-coupling Λ -hypernuclear shell-model wavefunctions in terms of the corresponding nuclear-core g.s. leading SU(4) supermultiplet components, except for $A = 8$ where the first excited nuclear-core level had to be included. This proved to be a sound and useful approximation, yielding $\Lambda\Sigma$ strong-interaction contributions close to those given in Figs. 1–3 of Ref. [24].³ Details will be given elsewhere. The listed $A = 7 - 10$ values of ΔV_{YN} exhibit strong SU(4) correlations, marked in particular by the enhanced value of 119 keV for the SU(4) nucleon-hole configuration in ${}^8_\Lambda\text{Be}-{}^8_\Lambda\text{Li}$ with respect to the modest value of 17 keV for the SU(4) nucleon-particle configuration in ${}^{10}_\Lambda\text{B}-{}^{10}_\Lambda\text{Be}$. This enhancement follows from the relative magnitudes of the Fermi-like interaction term $\bar{V}_{\Lambda\Sigma}^{0p}$ and its Gamow-Teller partner term $\Delta_{\Lambda\Sigma}^{0p}$ in Eq. (9). Noting that both $A = 4$ and $A = 8$ mirror hypernuclei correspond to SU(4) nucleon-hole configuration, the roughly factor two ratio of $\Delta V_{YN}(A = 4) = 232$ keV to $\Delta V_{YN}(A = 8) = 119$ keV reflects the approximate factor of two for the ratio between s -shell to p -shell $\Lambda\Sigma$ matrix elements, as discussed following Eq. (9).

Comparing $\Delta B_\Lambda^{\text{calc}}$ with $\Delta B_\Lambda^{\text{exp}}$ in Table 3, we note the reasonable agreement reached between the present $(\Lambda\Sigma)_e$ coupling model calculation and experiment for all four pairs of p -shell hypernuclei, $A = 7 - 10$, considered

³I am indebted to John Millener for providing me with some of the wavefunctions required here.

in this work. Extrapolating to heavier hypernuclei, one might naively expect negative values of $\Delta B_{\Lambda}^{\text{calc}}$, as suggested by the listed $A = 9, 10$ values. However, this rests on the assumption that the negative ΔV_C^{Λ} contribution remains as large upon increasing A as it is in the beginning of the p shell, which need not be the case. As nuclear cores beyond $A = 9$ become more tightly bound, the Λ hyperon is unlikely to compress these nuclear cores as much as it does in lighter hypernuclei, so that the additional Coulomb repulsion in ${}^{12}_{\Lambda}\text{C}$, for example, over that in ${}^{12}_{\Lambda}\text{B}$, while still negative, may not be sufficiently large to offset the attractive CSB contribution. In making this argument we rely on the expectation, based on SU(4) supermultiplet fragmentation patterns in the p shell, that ΔV_{YN} does not exceed ~ 100 keV.

Before closing the discussion of CSB in p -shell hypernuclei, we wish to draw attention to the state dependence of CSB splittings, recalling the vast difference between the calculated $\Delta B_{\Lambda}^4(0_{\text{g.s.}}^+)$ and $\Delta B_{\Lambda}^4(1_{\text{exc}}^+)$ in the s shell. In Table 4 we list CSB contributions $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ to several g.s. doublet excitation energies, as well the excitation energies ΔE^{CS} calculated by Millener [24] using charge symmetric (CS) YN spin-dependent interactions, including CS $\Lambda\Sigma$ contributions $\Delta E_{\Lambda\Sigma}^{\text{CS}}$ (also listed). It is tacitly assumed that ΔV_C^{Λ} is state independent for the hypernuclear g.s. doublet members. As for the other, considerably smaller contributions, we checked that ΔV_C^{Σ} remains at the 1 keV level and that the difference between the appropriate Σ -dominated ΔT_{YN} values is less than 10 keV. Under these circumstances, it is sufficient to limit the discussion to the state dependence of ΔV_{YN} alone, although the splittings $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ listed in the table include these other tiny contributions.

Table 4: Ground-state doublet splittings in several p -shell hypernuclei: ΔE^{exp} , ΔE^{CS} from [24] using CS spin-dependent YN interactions that include CS $\Lambda\Sigma$ contributions $\Delta E_{\Lambda\Sigma}^{\text{CS}}$, and CSB $\Lambda\Sigma$ contributions $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ considered in the present work, all in keV.

${}^A_{\Lambda}Z$	J_{exc}^{π}	$J_{\text{g.s.}}^{\pi}$	ΔE^{exp}	ΔE^{CS} [24]	$\Delta E_{\Lambda\Sigma}^{\text{CS}}$ [24]	$\Delta E_{\Lambda\Sigma}^{\text{CSB}}$
${}^8_{\Lambda}\text{Li}$	2^-	1^-	442 ± 2 [32]	445	149	-53.2
${}^9_{\Lambda}\text{Li}$	$\frac{5}{2}^+$	$\frac{3}{2}^+$	570 ± 120 [31]	590	116	+12.5
${}^{10}_{\Lambda}\text{B}$	2^-	1^-	< 100 [32, 33]	110	-10	-26.5

Inspection of Table 4 reveals that whereas CSB contributions $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ are negligible in ${}^9_{\Lambda}\text{Li}$, with respect to both $\Delta E_{\Lambda\Sigma}^{\text{CS}}$ and to the total CS splitting ΔE^{CS} , they need to be incorporated in re-evaluating the g.s. doublet splittings in ${}^8_{\Lambda}\text{Li}$ and in ${}^{10}_{\Lambda}\text{B}$.

- In ${}^8_\Lambda\text{Li}$, these $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ contributions spoil the perhaps fortuitous agreement between ΔE^{exp} , deduced from a tentative assignment of a γ -ray transition observed in the ${}^{10}\text{B}(K^-, \pi^-){}^{10}_\Lambda\text{B}$ reaction continuum spectrum [32], and ΔE^{CS} evaluated using the YN spin-dependent interaction parameters deduced from well identified γ -ray transitions in other hypernuclei. The 50 keV discrepancy arising from adding $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ surpasses significantly the typical 20 keV theoretical uncertainty in fitting doublet splittings in p -shell hypernuclei (see Table 1, Ref. [24]).
- The inclusion of $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ in the calculated ${}^{10}_\Lambda\text{B}$ g.s. doublet splitting helps solving the longstanding puzzle of not observing the $2_{\text{exc}}^- \rightarrow 1_{\text{g.s.}}^-$ γ -ray transition, thereby placing an upper limit of 100 keV on this transition energy [32, 33]. Including our CSB calculated contribution would indeed lower the expected transition energy from 110 keV to about 85 keV, in accordance with the experimental upper limit.⁴

It might appear unnatural that $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ is calculated to be a sizable fraction of $\Delta E_{\Lambda\Sigma}^{\text{CS}}$ in ${}^8_\Lambda\text{Li}$, or even exceed it in ${}^{10}_\Lambda\text{B}$. This may be understood noting that the evaluation of $\Delta E_{\Lambda\Sigma}^{\text{CSB}}$ involves a CSB small parameter of ~ 0.03 , see Eq. (5), whereas the evaluation of $\Delta E_{\Lambda\Sigma}^{\text{CS}}$ involves a small parameter of $\sqrt{P_\Sigma}$ which is less than 0.05 for ${}^8_\Lambda\text{Li}$ and less than 0.025 for ${}^{10}_\Lambda\text{B}$ in our $(\Lambda\Sigma)_e$ coupling model, see Table 3.

5. Conclusion

It was shown in this work how a relatively large CSB contribution of order 250 keV arises in $(\Lambda\Sigma)$ coupling models based on Akaishi's central-interaction G -matrix calculations in s -shell hypernuclei [16, 17], coming close to the binding energy difference $B_\Lambda({}^4_\Lambda\text{He}) - B_\Lambda({}^4_\Lambda\text{H}) = 350 \pm 60$ keV deduced from emulsion studies [3]. It was also argued that the reason for most of the $YNNN$ coupled-channel calculations done so far to come out considerably behind, with 100 keV at most by using NSC97f, is that their $\Lambda\Sigma$ channel coupling is dominated by strong tensor interaction terms. In this sense, the

⁴Adding CSB is not essential in the ${}^{10}_\Lambda\text{B}$ $\alpha p \Lambda$ cluster-model calculation [23] that results in 80 keV g.s. doublet excitation using CS ΛN interactions. However, to do a good job on the level of ~ 20 keV, one needs to include α -breakup ΛN contributions which are missing in this calculation.

CSB-dominated large value of $\Delta B_{\Lambda}^4(0_{\text{g.s.}}^+)$ places a powerful constraint on the strong-interaction YN dynamics.

In spite of the schematic nature of the present $(\Lambda\Sigma)$ coupling model of the $A = 4$ hypernuclei, which undoubtedly does not match the high standards of solving coupled-channel four-body problems, this model has the invaluable advantage of enabling a fairly simple application to heavier hypernuclei, where it was shown to reproduce successfully the main CSB features as disclosed from the several measured binding energy differences in p -shell mirror hypernuclei. More quantitative work, particularly for the ${}^{12}_{\Lambda}\text{C}$ – ${}^{12}_{\Lambda}\text{B}$ mirror hypernuclei, has to be done in order to confirm the trends established here in the beginning of the p shell upon relying exclusively on data reported from emulsion studies. Although the required calculations are rather straightforward, a major obstacle in reaching unambiguous conclusions is the unavailability of alternative comprehensive and accurate measurements of g.s. binding energies in mirror hypernuclei that may replace the existing old emulsion data.

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Note added after publication

A recent LQCD work [34] evaluates the Σ^0 – Λ mixing matrix element $\langle\Sigma^0|\delta M|\Lambda\rangle$ to be smaller by about factor of 2 than the one used in the present work upon substituting PDG [35] baryon mass values in the Dalitz-Von Hippel [19] $\text{SU}(3)_f$ -symmetry expression (2) which we copy here,

$$\langle\Sigma^0|\delta M|\Lambda\rangle = \frac{1}{\sqrt{3}}(M_{\Sigma^0} - M_{\Sigma^+} + M_p - M_n). \quad (11)$$

The resolution of this apparent discrepancy is that this much smaller value of $\langle\Sigma^0|\delta M|\Lambda\rangle$ in Ref. [34] is obtainable simply by substituting LQCD-calculated mass differences in Eq. (11), particularly $M_n - M_p = 2.70$ MeV which is about twice larger than the PDG value 1.29 MeV and which comes with a negative sign in (11). We conclude that this LQCD determination of Σ^0 – Λ mixing is no better than their determination of the neutron-proton mass difference.

References

- [1] For a recent review see G.A. Miller, A.K. Opper, E.J. Stephenson, *Annu. Rev. Nucl. Part. Sci.* 56 (2006) 253.
- [2] R. Machleidt, H. Mütter, *Phys. Rev. C* 63 (2001) 034005.
- [3] D.H. Davis, *Nucl. Phys. A* 754 (2005) 3c.
- [4] H. Tamura, et al., *Nucl. Phys. A* 914 (2013) 99. I thank Professor Tamura for providing me with this figure.
- [5] A. Esser, et al. (MAMI A1 Collaboration), *Phys. Rev. Lett.* (submitted) arXiv:1501.06823.
- [6] M. Bedjidian, et al. (CERN-Lyon-Warsaw Collaboration), *Phys. Lett. B* 62 (1976) 467, 83 (1979) 252.
- [7] A. Bamberger, et al. (CERN-Heidelberg-Warsaw Collaboration), *Nucl. Phys. B* 60 (1973) 1.
- [8] J. Carlson, *AIP Conf. Proc.* 224 (1991) 198.
- [9] S.A. Coon, H.K. Han, J. Carlson, B.F. Gibson, in *Meson and Light Nuclei '98*, eds. J. Adams, et al. (WS, Singapore, 1999) pp.407-413, arXiv:nucl-th/9903034.
- [10] A. Nogga, H. Kamada, W. Glöckle, *Phys. Rev. Lett.* 88 (2002) 172501.
- [11] J. Haidenbauer, U.-G Meißner, A. Nogga, H. Polinder, *Lect. Notes Phys.* 724 (2007) 113.
- [12] A. Nogga, *Nucl. Phys. A* 914 (2013) 140, and references to earlier work cited therein.
- [13] Th.A. Rijken, V.G.J. Stoks, Y. Yamamoto, *Phys. Rev. C* 59 (1999) 21.
- [14] P.M.M. Maessen, Th.A. Rijken, J.J. de Swart, *Phys. Rev. C* 40 (1989) 2226.
- [15] E. Hiyama, *Nucl. Phys. A* 914 (2013) 130, and references cited therein.

- [16] Y. Akaishi, T. Harada, S. Shinmura, K.S. Myint, Phys. Rev. Lett. 84 (2000) 3539.
- [17] H. Nemura, Y. Akaishi, Y. Suzuki, Phys. Rev. Lett. 89 (2002) 142504.
- [18] D.J. Millener, Nucl. Phys. A 754 (2005) 48c, 804 (2008) 84.
- [19] R.H. Dalitz, F. Von Hippel, Phys. Lett. 10 (1964) 153. See also the Note added ahead of the References section in this arXiv version.
- [20] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, Y. Yamamoto, Phys. Rev. C 65 (2002) 011301(R).
- [21] J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G Meißner, A. Nogga, W. Weise, Nucl. Phys. A 915 (2013) 24.
- [22] E. Hiyama, Y. Yamamoto, T. Motoba, M. Kamimura, Phys. Rev. C 80 (2009) 054321.
- [23] Y. Zhang, E. Hiyama, Y. Yamamoto, Nucl. Phys. A 881 (2012) 288; E. Hiyama, Y. Yamamoto, Prog. Theor. Phys. 128 (2012) 105.
- [24] D.J. Millener, Nucl. Phys. A 881 (2012) 298, and references listed therein. For a recent application to neutron-rich hypernuclei, see Ref. [25].
- [25] A. Gal, D.J. Millener, Phys. Lett. B 725 (2013) 445.
- [26] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, Y. Yamamoto, Phys. Rev. C 66 (2002) 024007.
- [27] D.J. Millener, unpublished (March 2015).
- [28] H. Tamura, et al., Phys. Rev. Lett. 84 (2000) 5963.
- [29] S.N. Nakamura, et al. (HKS JLab E01-011 Collaboration), Phys. Rev. Lett. 110 (2013) 012502.
- [30] T. Gogami, Ph.D. thesis, Tohoku University, Sendai, Japan (2014).
- [31] G.M. Urciuoli, et al. (JLab Hall A Collaboration), Phys. Rev. C 91 (2015) 034308.

- [32] R.E. Chrien, et al., Phys. Rev. 41 (1990) 1062.
- [33] H. Tamura, et al., Nucl. Phys. A 754 (2005) 58c.
- [34] R. Horsley, et al. (QCDSF-UKQCD Collaboration), Phys. Rev. D 91 (2015) 074512.
- [35] K.A. Olive, et al. (PDG), Chin. Phys. C 38 (2014) 090001.